

Yule's "Nonsense Correlation" and New Frontiers in Statistical Inference for Stochastic Processes



Abstract: I will first speak about how we recently resolved a longstanding open question in applied probability. Consider the standard empirical correlation ρ_n , which is defined for two related series of data of length n using the standard Pearson correlation statistic. This empirical correlation is known as Yule's "nonsense correlation" in honor of the British statistician G. Udny Yule, who in 1926 described the phenomenon by which empirical correlation fails to gauge independence of data series for random walks and for other time series. For the case of two independent and identically distributed random walks independent from each other, Yule empirically observed that the distribution of the empirical correlation is not concentrated around 0; rather, it is "volatile" in the sense that its distribution is heavily dispersed and is frequently large in absolute value. This well-documented effect was ignored by many scientists over the decades, up to the present day, even sparking recent controversies in climate-change attribution. Since the 1960s, some probabilists have wanted to eliminate any possible ambiguity about the issue by computing the variance of the continuous-time version ρ of Yule's nonsense correlation, based on the paths of two independent Wiener processes. The problem would remain open until we finally closed it in our work entitled "Yule's 'nonsense correlation' solved!" (*The Annals of Statistics*, 2017). I will then turn to speaking about our subsequent success in explicitly calculating all moments of ρ for two independent Wiener processes. Our solution leads to the first approximation to the density of Yule's nonsense correlation. We are also able to explicitly compute higher moments of Yule's nonsense correlation when the two independent Wiener processes are replaced by two correlated Wiener processes, two independent Ornstein-Uhlenbeck processes, and two independent Brownian bridges. We then consider extending the definition of ρ to the time interval $[0, T]$ for any $T > 0$ and prove a Central Limit Theorem for the case of two independent Ornstein-Uhlenbeck processes. All of these aforementioned results appear in our preprint entitled "The distribution of Yule's nonsense correlation" (presently under invited revision, and available via <https://arxiv.org/pdf/1909.02546.pdf>). I will conclude with some concrete applications of our work to the study of weather and climate extremes. In particular, one pressing question of interest to our collaboration with the U.S. Office of Naval Research (ONR) is as follows: can the frequency of open ocean "freak wave" events observed by sea-faring vessels be correlated to and/or predicted and/or forecasted by sea-level rise and/or coastal wind extremes? This answer to this question should help inform the evaluation of risk exposure for U.S. Navy installations and vessels, particularly those based on the Atlantic seaboard.

Bio: Philip Ernst is an Associate Professor with tenure at Rice University. Ernst earned his Ph.D. in statistics from the Wharton School of the University of Pennsylvania in 2014, and joined the Rice faculty that same year. His research interests include exact distribution theory, mathematical finance, operations research, optimal stopping, queueing systems, statistical inference for stochastic processes, and stochastic control. He is an associate editor for *Mathematics of Operations Research*, an associate editor for *Stochastics*, an associate editor for *Statistics and Probability Letters*, and an associate editor of *Journal of Stochastic Analysis*. He is also Guest Editor-in-Chief of "In Memoriam: Larry Shepp," a special issue of *Stochastic Processes and their Applications* to appear in Spring 2022. Ernst's research is funded by the U.S. Office of Naval Research (ONR), U.S. Army Research Office (ARO), and the National Science Foundation (NSF). He is the recipient of numerous international and national research awards. In 2020, Ernst received the (inaugural) INFORMS Donald P. Gaver, Jr. Early Career Award for Excellence in Operations Research. In 2018, Ernst was honored with the Tweedie New Researcher Award from the Institute of Mathematical Statistics, widely considered the highest honor for excellence in research for an early-career mathematical statistician or applied probabilist. In that same year, Ernst also received the prestigious Army Research (ARO) Young Investigator Award. He has also won five teaching awards from Rice University.

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Zoom/Online